

Prove the reduction formula $\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$ (where $n \neq 1$).

SCORE: ____ / 6 PTS

NOTE: You must show how to get this formula.

You will receive 0 credit if your "proof" is differentiating both sides of the equation.

$$\begin{array}{l} \frac{u}{\sec^{n-2} u} \\ \frac{dv}{\sec^2 u} \\ \Rightarrow \tan u \end{array}$$

$$\int \sec^n u \, du = \sec^{n-2} u \tan u - (n-2) \int \sec^{n-2} u \tan^2 u \, du$$

$$\int \sec^n u \, du = \sec^{n-2} u \tan u - (n-2) \int \sec^{n-2} u (\sec^2 u - 1) \, du$$

$$\int \sec^n u \, du = \sec^{n-2} u \tan u - (n-2) \int \sec^n u \, du + (n-2) \int \sec^{n-2} u \, du$$

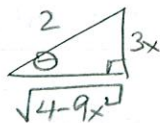
$$(n-1) \int \sec^n u \, du = \sec^{n-2} u \tan u + (n-2) \int \sec^{n-2} u \, du$$

$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

Evaluate $\int x^2 \sqrt{4-9x^2} dx$.

SCORE: ____ / 9 PTS

$$x = \frac{2}{3} \sin \theta \longrightarrow \sin \theta = \frac{3x}{2}$$
$$dx = \frac{2}{3} \cos \theta d\theta$$



$$\int \frac{4}{9} \sin^2 \theta \cdot 2 \cos \theta \cdot \frac{2}{3} \cos \theta d\theta$$

① POINT EACH
UNLESS OTHERWISE
MARKED

$$= \frac{16}{27} \int \sin^2 \theta \cos^2 \theta d\theta$$

$$= \frac{16}{27} \int (1 - \cos^2 \theta) \cos^2 \theta d\theta$$

$$= \frac{16}{27} \left[\int \cos^2 \theta d\theta - \int \cos^4 \theta d\theta \right] \star \text{TALK TO ME IF YOU WROTE IN TERMS OF SIN X}$$

$$= \frac{16}{27} \left[\int \cos^2 \theta d\theta - \frac{1}{4} \cos^3 \theta \sin \theta - \frac{3}{4} \int \cos^2 \theta d\theta \right]$$

$$= \frac{16}{27} \left[-\frac{1}{4} \cos^3 \theta \sin \theta + \frac{1}{4} \left(\frac{1}{2} \cos \theta \sin \theta + \frac{1}{2} \theta \right) \right] + C$$

$$= \frac{-4}{27} \left(\frac{\sqrt{4-9x^2}}{2} \right)^3 \frac{3x}{2} + \frac{2}{27} \frac{\sqrt{4-9x^2}}{2} \frac{3x}{2} + \frac{2}{27} \sin^{-1} \frac{3x}{2} + C$$

$$= \frac{-1}{36} \times \sqrt{4-9x^2}^3 + \frac{1}{18} \times \sqrt{4-9x^2} + \frac{2}{27} \sin^{-1} \frac{3x}{2} + C \longleftarrow \text{② IF YOU FORGOT}$$

Evaluate $\int \tan^4 x \sec^4 x dx = \int \tan^4 x \sec^2 x \cdot \sec^2 x dx$

SCORE: ____ / 5 PTS

① $u = \tan x$
 $du = \sec^2 x dx$

$= \int u^4 (u^2 + 1) du$ ②

$= \int (u^6 + u^4) du$

$= \frac{1}{7} u^7 + \frac{1}{5} u^5 + C$

$= \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$

① $-\frac{1}{2}$ IF YOU FORGOT

★ TALK TO ME IF YOU WROTE IN TERMS OF SEC X AND USED REDUCTION FORMULA

Evaluate $\int \frac{(\ln x)^2}{\sqrt{x}} dx = \underbrace{2\sqrt{x}(\ln x)^2}_{\textcircled{1/2}} - \underbrace{8\sqrt{x}\ln x}_{\textcircled{2}} + \underbrace{16\sqrt{x}}_{\textcircled{1/2}} + C$

SCORE: ____ / 5 PTS

$$\begin{array}{r} \frac{u}{(\ln x)^2} + \frac{dv}{x^{-1/2}} \\ \frac{2\ln x}{x} \quad 2x^{1/2} \\ \hline x \quad \frac{1}{x} \\ 2\ln x \quad 2x^{-1/2} \\ \hline \frac{2}{x} \quad 4x^{1/2} \\ x \quad \frac{1}{x} \\ 2 \quad 4x^{-1/2} \\ 0 \quad 8x^{1/2} \end{array}$$

$\textcircled{1/2}$ IF YOU FORGOT

Evaluate $\int (x^3 - 2x^2 - 3)e^{-\frac{x}{2}} dx$. = $\underline{-2(x^3 - 2x^2 - 3)e^{-\frac{x}{2}}}$

SCORE: ____ / 5 PTS

<u>u</u>		<u>dv</u>	
$x^3 - 2x^2 - 3$	+	$e^{-\frac{x}{2}}$	
$3x^2 - 4x$	-	$-2e^{-\frac{x}{2}}$	
$6x - 4$	+	$4e^{-\frac{x}{2}}$	
6	-	$-8e^{-\frac{x}{2}}$	
0		$16e^{-\frac{x}{2}}$	

$\underline{-4(3x^2 - 4x)e^{-\frac{x}{2}}}$

$\underline{-8(6x - 4)e^{-\frac{x}{2}}}$

$\underline{-96e^{-\frac{x}{2}}}$

① POINT EACH

= $\underline{(-2x^3 - 8x^2 - 32x - 58)e^{-\frac{x}{2}}} + C$

↖ $\textcircled{-\frac{1}{2}}$ IF YOU FORGOT